Abstracts in alphabetical order by speaker's name

in Workshop "Number Theory and Ergodic Theory"
from Feb. 8, 2020 through Feb. 10, 2020.

Shigeki Akiyama (Univerisity of Tsukuba)

"A geometric characterization of pure discrete symbolic dynamics"

In a certain family of symbolic dynamical systems (one-sided or two-sided), there is a well-known geometric realization due to Rauzy and Arnoux-Ito to construct conjugacy to domain exchange via broken lines. We axiomize this idea and give a short direct way to find metrical isomorphism to torus rotation. If everything works fine, the original system has pure discrete spectrum. We can also prove that this method is "if and only if" for irreducible Pisot unit substitution. Important feature of this new formulation is that all computation boils down to automata computation. Finally we give an S-adic system on 3 letters having pure discrete spectrum. This is a joint work with Paul Mercat.

Keywords: Pisot Substitution, pure discrete system, S-adic system.

Hiroshi Fujisaki (Kanazawa University)

"A simple construction of the full-length binary sequences based on the discretized Markov β-transformations and their correlational properties"

We have previously defined the discretized Markov transformations and the full-length sequences based on such transformations. De Bruijn sequences can be regarded as the full-length sequences based on the discretized Markov β-transformation with $\beta = 2$. Recently, Sawada et al. proposed an efficient construction of de Bruijn sequence. We modify their construction and apply it to construct a full-length sequence based on the discretized Markov β-transformation, where β is the golden mean. We also give correlational properties of not only de Bruijn sequences constructed by Sawada et al. but the full-length sequences constructed in this research, which are based on the discretized golden mean transformation.

Keywords: discretized Markov transformation, β-transformation, full-length sequence, necklace, correlation function.

Yu Ito (Kyoto Sangyo University)

"Resolution of sigma-fields for multiparticle finite-state action evolutions with infinite past"

For multiparticle finite-state action evolutions, we report that the observation sigma-fields are proven to have a resolution consisting of a reduced driving noise, the remote past noise and a third noise. The general theory of infinite convolutions on finite semigroups is utilized in our proofs. This talk is based on joint work with Toru Sera and Kouji Yano.

Keywords: Resolution of sigma-fields, Stochastic recursive equation, Rees decomposition, Infinite convolution

Shoichi Kamada (Tokyo Metropolitan University)

"On failure probabilities of reductions from subset sum problems to lattice problems, and multifractal analysis"

Let $\{a_1, \ldots, a_s\}$ be a set of positive integers. Then an integer C is representable as subset sum if $C = x_1a_1 + \cdots + x_sa_s$, where $x_i \in \{0, 1\}$ for $i = 1, \ldots, s$. The subset sum problem is to find such x_i's. This problem is important in cryptography. For example, knapsack cryptosystems are based on subset sum problems.

In the context of knapsack cryptography, failure probabilities of reductions from subset sum problems to lattice problems play important roles in low-density attacks. The density in this context was first introduced by Lagarias and Odlyzko, and several researchers investigated some properties on this density. The density is a good parameter to investigate upper bounds of failure probabilities of reductions.

In this talk, we give lower bounds and numerical estimates for these failure probabilities of reductions, and we connect them with multifractal analysis via some quantity related to representability of integers as subset sum.

Keywords: subset sum problem, lattice problem, shortest vector problem, knapsack cryptography, density, representability of integers as subset sums, additive structures of integers, combinatorics.

Teturo Kamae (Osaka City University)

"Non-correlated pattern sequences"

We consider infinite sequences $x = x_0x_1 \ldots$, where x_n is defined by counting patterns in a given set appearing in the binary representation of n so that $x_n = 1$ or -1 corresponds to this number being even or odd. They are called pattern sequences. For example, the Thue-Morse sequence is the pattern sequence with pattern set $\{1\}$, while the Rudin-Shapiro sequence is with pattern set $\{11\}$. We give a sufficient condition for the pattern sequences to be non-correlated.

Keywords: Non-correlated binary sequences, Generalization of the Rudin-Shapiro sequence

Hajime Kaneko (University of Tsukuba)

"Hensel's lemma and application for the base-b expansions of integers"

Hensel's lemma implies the existence and uniqueness of the zeros of polynomials under certain assumptions. Axelsson and Khrennikov generalized Hensel's lemma for Lipschitz functions defined on the ring $\mathbf{Z}_p$ of p-adic integers. They asked for a generalization of Hensel's lemma to general continuous function. In this talk, we give an answer for this problem. Moreover, we consider application for the base-b expansions of integers. This is a joint work with Thomas Stoll.

Keywords: Hensel's lemma, van der Put series, base-b expansion.

Dong Han Kim (Dongguk University)

"Intrinsic Diophantine approximation of the sphere"

In this talk we consider the intrinsic Diophantine approximation on the sphere by its rational points. We give complete descriptions of discrete parts of the Lagrange spectra, arising from intrinsically approximating points on a circle by the rational points on it. We also present some relevant results for a 2-sphere.

Keywords: Lagrange spectrum, Markov spectrum, Diophantine approximation.

Yuto Nakajima (Kyoto University)

"Slicing the fractal imaginary cubes"

We investigate the Hausdorff dimension of the intersection of the N-dimensional Sierpiński gasket (we denote by SG^N this fractal) with a hyperplane in a fixed direction. The situation for the case $N = 2$ has been studied extensively by I. Benjamini and Y. Peres (1991). We calculate the Hausdorff dimension of intersections of SG^N with hyperplanes by using the theory of non-autonomous conformal iterated function system which is found in L. Rempe-Gillen and Mariusz Urbański's paper in 2016. In the proof, we use the uniqueness of binary digits expansion of non-dyadic real numbers.

Furthermore, we consider the slicing the fractal imaginary cubes, which are the generalization of SG^3. Fractal imaginary cube A is a subset of three-dimensional Euclidean space generated by an iterated function system Φ which satisfies the following three conditions (H. Tsuiki (2007)). There is some cube C such that the projections of A in the directions of the faces of C equal to these projections of C. The Hausdorff dimension of A is equal to 2. The similarity transformations of the IFS Φ do not include rotational parts.

N 次元シエルピンスキーガスケットの断面について考える。特にある方向を固定し、それに垂直な断面のハウスドルフ次元を考える。$N = 2$ の時には I. Benjamini 氏と Y. Peres 氏により広範囲に研究されている (1991 年)。我々はより一般的の次元のシエルピンスキーガスケットの断面のハウスドルフ次元を 2016 年に L. Rempe-Gillen 氏と Mariusz Urbański 氏により構築された非自励的等角反復関数系の理論を用いることにより計算する。その証明では二進展開の一意性を用いる。さらに三次元シエルピンスキーガスケットの一般化であるフラクタルイマジナリーキューブの断面について考える。フラクタルイマジナリーキューブは立木秀樹氏により考案された以下の三条件を満たす相似写像からなる反復関数系の極限集合である。まず、ある立方体が存在して面の方向に射影した時にできる影がその立方体の影と同じである。そしてハウスドルフ次元が 2 である。最後に反復関数系の相似写像は回転項を含まない。

Keywords: fractal, non-autonomous conformal iterated function system, Hausdorff dimension.

Ryotaro Okazaki (The university of Tokyo)

"Using Number Theory Software Sagemath"

We will demonstrate how Sagemath helps a research project in number theory.

数論ソフト Sagemath の数論の研究プロジェクトでの活用の仕方を見ていきます。

Keywords: Sagemath, Maxima Pari-GP, Python, LaTeX, Markdown.

Kota Saito (Nagoya University)

"Szemerédi's theorem and fractal dimensions of sets not containing weak arithmetic progressions"

In this talk, we give new lower and upper bounds for fractal dimensions of sets which do not contain (k, ϵ)-arithmetic progressions (APs). More precisely, we say that a subset F of real numbers does not contain (k, ϵ)-APs if one cannot find any APs of length k with gap difference Δ in the $\epsilon\Delta$-neighborhood of F. The goal of this talk is to show that upper and lower bounds for the Assouad and Hausdorff dimensions of sets which do not contain (k, ϵ)-APs can be written in terms of $r_k(1/\epsilon)$. Here $r_k(N)$ denotes the largest cardinality of subsets of $\{1, \dots, N\}$ which do not contain any APs of length k. Moreover, we find equivalent conditions between Szemerédi's theorem and bounds for fractal dimensions of sets which do not contain (k, ϵ)-APs.

Keywords: Szemerédi's theorem, Arithmetic progressions, Assouad dimension, Hausdorff dimension, Self-similar set.

Toru Sera (Kyoto University)

"A conditional limit theorem for the Pomeau–Manneville map"

Pomeau–Manneville map is an interval map with an indifferent fixed point. Its orbit spends most of its time in the neighborhood of the point. Thanks to Darling–Kac type limit theorem, we can understand the amount of time which the orbit spends away from the point. In this talk, I present a conditional version of Darling–Kac type limit theorem under the condition that the orbit is away from the point at the end of the observation. This talk is based on joint work with Jon Aaronson (Tel Aviv University).

Keywords: Intermittent dynamical system, Darling–Kac limit theorem.

Naoto Shimaru (Okayama University of Science)

"On behaviors of irrational rotations"

We develop the idea of Mori and Takashima (On the distribution of leading digits of a^n: a study via χ^2 statistics, Periodica Math. Hungr., 73 (2016) 224– 239.) and we introduce its refinement, "rational rotation approximation". We consider not only the rational rotation approximation but also the Ostrowski expansion and we study important behaviors of irrational rotations, such as the position of point of the irrational rotation itself, the behaviors of partial sums of irrational rotations and the behaviors of discrepancies of irrational rotations. In case that an irrational number α has large partial quotients, our methods are very useful for studies of irrational rotations and we give some formulas and estimates for the above behaviors. Furthermore, we give the mathematical explanations for the unusual behaviors of irrational rotations based on such α.

Keywords: Irrational rotation, Continued fraction, Discrepancy.

Shintaro Suzuki (Keio Institute of Pure and Applied Sciences)
"The set of conjugates of all Yrrap numbers"

A real number $\beta > 1$ is called a Parry number if the corresponding β-transformation has a finite orbit at 1. The same analogue to the negative β-transformation is called a Yrrap number. In this talk, we determine the closure of the set of conjugates of Yrrap numbers and compare it with that of Parry numbers. As an application, we give countably many Parry numbers which are non-Yrrap numbers, and vice versa.

Keywords: Ergodic theory, negative beta-transformations, Yrrap numbers.

Hiroki Takahasi (KiPAS, Keio University)
"Large deviation principle for arithmetic functions in the backward continued fraction expansion"

We show the large deviation principle for arithmetic functions in the backward continued fraction expansion, and analyze the structure of minimizers of the late function.

Keywords: large deviation principle, backward continued fraction, minimizer.

Masato Takei (Yokohama National University)
"Probabilistic analysis of Takagi class functions: Rate of convergence"

We consider a generalized version of the Takagi function, which is one of the most famous example of nowhere differentiable continuous functions. We investigate a set of conditions to describe the rate of convergence of Takagi class functions from the probabilistic point of view: The law of large numbers, the central limit theorem, and the law of the iterated logarithm. On the other hand, we show that the Takagi function itself does not satisfy the law of large numbers in the usual sense. (Based on a joint work with Shoto Osaka.)

至るところ微分不可能な連続関数の有名な例である高木関数の定義を一般化した高木クラスの連続関数について，確率論的な見地から調べる。関数を定める係数が大まかにいって指数関数よりも緩やかに減少する場合には，典型的な点における極限関数への収束の速さを記述する極限定理 (大数の法則，中心極限定理，重複対数の法則) が得られる。一方，元祖の高木関数を含む，指数的に減少する係数を与えた場合には通常の意味合いでは大数の法則が成立しない。(大坂翔人氏との共同研究に基づく．)

Keywords: Nowhere differentiable continuous functions, Limit theorems.

Jun-ichi Tamura (Tsuda College)

"Some problems and results around Pentagonal Number Theorem of Euler, and Moonshine, etc."

Let $R \geq 2$ be an integer, and $a_n = a_n(R)$ be a linear recurrence sequence defined by

$$\begin{aligned} a_n &= 2^{n-1} \quad (1 \leq n \leq R), \\ a_n &= a_{n-1} + a_{n-2} + \cdots + a_{n-R} \quad (n \geq R+1). \end{aligned}$$

We give a necessary and sufficient condition for the set of all the coefficients of the polynomial

$$P_n(z) := \prod_{1 \leq m \leq n} (1 + qz^{a_m})$$

being a subset of $\{\pm q^n \mid 0 \leq n \leq J-1\} \cup \{0\}$ for all $0 \leq n \leq kR$, where q is a primitive J-th root of unity, and k is a positive integer. We shall mention some curious

phenomena/results/conjectures/problems

concerning

diophantine equations, transcendence, continued fractions,
prime numbers, Moonshine^2, music

related to such finite/infinite products.

Keywords: finite/infinite products, continued fractions, transcendence, cuorious phenomena.

Kenichiro Yamamoto (Nagaoka University of Technology)

"Topological entropy of the set of generic points for (α -β)-shifts"

We prove that all (α-β)-shifts with $0 \leq \alpha < 1$ and $\beta > 2$ are saturated, that is, for any invariant measure, the topological entropy of the set of generic points coincides with the metric entropy.

Keywords: topological entropy, generic points, (α - β)-shifts.