

$$\lim_{x \rightarrow \infty} x(\sqrt{x^2 + 1} - x)$$

$$= \lim_{x \rightarrow \infty} \frac{x(\sqrt{x^2 + 1} - x)(\sqrt{x^2 + 1} + x)}{\sqrt{x^2 + 1} + x}$$

分母・分子を $(\sqrt{x^2 + 1} + x)$ 倍

$$= \lim_{x \rightarrow \infty} \frac{x(x^2 + 1 - x^2)}{\sqrt{x^2 + 1} + x}$$

$$\lim_{x \rightarrow \infty} x(\sqrt{x^2 + 1} - x) = \lim_{x \rightarrow \infty} \frac{x}{\sqrt{x^2 + 1} + x}$$

$$= \lim_{x \rightarrow \infty} \frac{1}{\sqrt{1 + \frac{1}{x^2}} + 1}$$

$$= \frac{1}{2}$$

分母・分子を $\frac{1}{x}$ 倍