

分母・分子を有理化する． まず分子は，

$$\begin{aligned}\sqrt{x+3} - \sqrt{x} &= \frac{(\sqrt{x+3} - \sqrt{x})(\sqrt{x+3} + \sqrt{x})}{\sqrt{x+3} + \sqrt{x}} \\ &= \frac{3}{\sqrt{x+3} + \sqrt{x}}\end{aligned}$$

次に分母は,

$$\begin{aligned} & \frac{1}{\sqrt{x+2} - \sqrt{x+1}} \\ &= \frac{\sqrt{x+2} + \sqrt{x+1}}{(\sqrt{x+2} - \sqrt{x+1})(\sqrt{x+2} + \sqrt{x+1})} \\ &= \sqrt{x+2} + \sqrt{x+1} \end{aligned}$$

$$\lim_{x \rightarrow \infty} \frac{\sqrt{x+3} - \sqrt{x}}{\sqrt{x+2} - \sqrt{x+1}} = \lim_{x \rightarrow \infty} \frac{3(\sqrt{x+2} + \sqrt{x+1})}{\sqrt{x+3} + \sqrt{x}}$$

$$= \lim_{x \rightarrow \infty} \frac{3 \left(\sqrt{1 + \frac{2}{x}} + \sqrt{1 + \frac{1}{x}} \right)}{\sqrt{1 + \frac{3}{x}} + 1}$$

分母・分子を $\frac{1}{\sqrt{x}}$ 倍

$$= 3$$