

(1) $f'(x) = \frac{1}{x}$ であるので $f'(1) = 1$

したがって

$$\begin{aligned} f'(1) &= \lim_{x \rightarrow 0} \frac{f(x+1) - f(1)}{x} \\ &= \lim_{x \rightarrow 0} \frac{\log(1+x)}{x} = 1 \end{aligned}$$

一方

$$\begin{aligned} &= \lim_{x \rightarrow 0} (1+x)^{\frac{1}{x}} = \lim_{x \rightarrow 0} e^{\log(1+x)^{\frac{1}{x}}} \\ &= \lim_{x \rightarrow 0} e^{\frac{\log(1+x)}{x}} = e \end{aligned}$$