

加法定理

$$\cos(m \pm n)x = \cos mx \cos nx \mp \sin mx \sin nx$$

辺々加えて

$$\cos mx \cos nx = \frac{1}{2}(\cos(m+n)x + \cos(m-n)x).$$

これを積分

まず $m \neq n$ なら

$$\begin{aligned} I &= \frac{1}{2} \int (\cos(m+n)x + \cos(m-n)x) dx \\ &= \frac{1}{2} \left(\frac{\sin(m+n)x}{m+n} + \frac{\sin(m-n)x}{m-n} \right) + C. \end{aligned}$$

また $m = n$ なら

$$\begin{aligned} I &= \frac{1}{2} \int (\cos 2x + 1) dx \\ &= \frac{1}{4n} \sin 2nx + \frac{1}{2}x + C. \end{aligned}$$