

$$x^2 + 1 = \tan^2 x + 1 = \frac{1}{\cos^2 t},$$

$$\frac{dx}{dt} = \frac{1}{\cos^2 t}$$

$$\begin{aligned} I &= \int \frac{1/\cos^2 t}{1/\cos^4 t} dt = \int \cos^2 t dt \\ &= \int \frac{\cos 2t + 1}{2} dt \\ &= \frac{\sin 2t}{4} + \frac{t}{2} + C = \frac{\sin t \cos t}{2} + \frac{t}{2} + C \end{aligned}$$

$$\begin{aligned} &= \frac{\tan t}{1/\cos^2 t} + \frac{t}{2} + C \\ &= \frac{1}{2} \left(\frac{x}{x^2 + 1} + \tan^{-1} x \right) + C \end{aligned}$$