

$$t = x + \sqrt{x^2 + 3} \text{ とおく}$$

$$x = \frac{t^2 - 3}{2t}, \quad \sqrt{x^2 + 3} = \frac{t^2 + 3}{2t}$$

$$\frac{dx}{dt} = \frac{t^2 + 3}{2t^2}$$

$$\begin{aligned} I &= \int \frac{2t}{t^2 - 3} \frac{2t}{t^2 + 3} \frac{t^2 + 3}{2t^2} \\ &= 2 \int \frac{dt}{t^2 - 3} \\ &= \frac{1}{\sqrt{3}} \int \left(\frac{1}{t - \sqrt{3}} - \frac{1}{t + \sqrt{3}} \right) dt \end{aligned}$$

$$= \frac{1}{\sqrt{3}} \log \left| \frac{t - \sqrt{3}}{t + \sqrt{3}} \right| + C$$

$$= \frac{1}{\sqrt{3}} \log \left| \frac{x + \sqrt{x^2 + 3} - \sqrt{3}}{x + \sqrt{x^2 + 3} + \sqrt{3}} \right| + C$$