

$t = \tan \frac{x}{2}$ とおく

$$\cos x = \frac{1 - t^2}{1 + t^2}$$

$$\frac{dx}{dt} = \frac{2}{t^2 + 1}$$

$$\begin{aligned} I &= \int \frac{\frac{2}{t^2+1} dt}{2\frac{1-t^2}{1+t^2} + 1} = 2 \int \frac{dt}{3-t^2} \\ &= \frac{1}{\sqrt{3}} \int \left(\frac{1}{t+\sqrt{3}} - \frac{1}{t-\sqrt{3}} \right) dt \end{aligned}$$

$$\begin{aligned} &= \frac{1}{\sqrt{3}} \log \left| \frac{t + \sqrt{3}}{t - \sqrt{3}} \right| + C \\ &= \frac{1}{\sqrt{3}} \log \left| \frac{\tan \frac{x}{2} + \sqrt{3}}{\tan \frac{x}{2} - \sqrt{3}} \right| + C \end{aligned}$$