104 ページ	章末問題 5.1(4)	解答	1/3

$\dfrac{x}{t} = u$ とおく.

同次形

$$x = tu,\ \frac{dx}{dt} = u + t\frac{du}{dt}$$

$$u + t\frac{du}{dt} = 2u + \frac{1}{u}$$

$$t\frac{du}{dt} = u + \frac{1}{u} = \frac{u^2+1}{u}$$

$$\int \frac{u}{u^2+1}\,du = \int \frac{dt}{t}$$

左辺は置換積分
$u^2+1 = v$ とおくと
$2udu = dv$

$$\frac{1}{2}\log(u^2+1) = \log|t| + C$$

$$\log(u^2+1) - 2\log|t| = 2C$$

$\pm e^{2C}$ を改めて C とおく

$$\frac{u^2+1}{t^2} = C$$

$$\therefore \quad \frac{x^2}{t^2} + 1 = Ct^2$$

(C は任意定数)