104ページ	章末問題5.6	解答	1/2

(1) $x = y + 2,\ \dfrac{dx}{dt} = \dfrac{dy}{dt}$ を方程式に代入

$$\frac{dy}{dt} = \frac{y + 2 + t - 2}{y + 2 - t - 2} = \frac{y + t}{y - t}$$

よって

$$\frac{dy}{dt} = \frac{y + t}{y - t}$$

(2) 右辺の分母分子を t で割ると

$$\frac{dx}{dt} = \frac{\dfrac{y}{t} + 1}{\dfrac{y}{t} - 1}$$

同次形

$\dfrac{y}{t} = u$ とおくと　$\dfrac{dy}{dt} = t\dfrac{du}{dt} + u$

104ページ	章末問題5.6	解答	3/2

$$t\frac{du}{dt} + u = \frac{u+1}{u-1}$$

$$t\frac{du}{dt} = \frac{u+1}{u-1} - u = \frac{-u^2+2u+1}{u-1}$$

$$\frac{u-1}{u^2-2u-1}\frac{du}{dt} = -\frac{1}{t}$$

$$\int \frac{u-1}{u^2-2u-1}\,du = -\int \frac{dt}{t}$$

104ページ	章末問題5.6	解答	4/2

$u^2 - 2u - 1 = v$ とおくと　$2(u-1)du = dv$

$$\frac{1}{2}\int \frac{dv}{v} = -\int \frac{dt}{t}$$

$$\frac{1}{2}\log|v| = -\log|t| + C$$

$$\log|v| + 2\log|t| = 2C$$

$$t^2 v = \pm e^{2C} = C$$

$\pm C$ を改めて C とおいた

104ページ	章末問題5.6	解答	5/2

$$t^2(u^2 - 2u - 1) = C$$

$$t^2\left\{\left(\frac{y}{t}\right)^2 - 2\frac{y}{t} - 1\right\} = C$$

$$y^2 - 2ty - t^2 = C$$

$$\therefore \quad (x-2)^2 - 2t(x-2) - t^2 = C$$