126 ページ	章末問題 6.1(3)	解答	1/3

$u = \dfrac{x}{y}$ とおく。

$$
\begin{aligned}
z_x &= (\tan^{-1} u)' \left(\frac{x}{y}\right)_x = \frac{1}{1+u^2}\frac{1}{y} = \frac{y}{x^2+y^2} \\
z_y &= (\tan^{-1} u)' \left(\frac{x}{y}\right)_y = \frac{1}{1+u^2}\left(-\frac{x}{y^2}\right) = -\frac{x}{x^2+y^2}
\end{aligned}
$$

$$
\begin{aligned}
z_{xx} &= -\frac{y(x^2+y^2)_x}{(x^2+y^2)^2} = -\frac{2xy}{(x^2+y^2)^2} \\
z_{yy} &= \frac{y(x^2+y^2)_y}{(x^2+y^2)^2} = \frac{2xy}{(x^2+y^2)^2}
\end{aligned}
$$

$$
\begin{aligned}
z_{xy} &= -\frac{(y)_y(x^2+y^2) - y(x^2+y^2)_y}{(x^2+y^2)^2} \\
&= \frac{x^2+y^2-2y^2}{(x^2+y^2)^2} = \frac{x^2-y^2}{(x^2+y^2)^2} \\
z_{yx} &= z_{xy}
\end{aligned}
$$