$u = \dfrac{x}{y}$ とおいて，合成関数の微分法を使うと，

108 ページ 例題 6.1 (2) の解答を参考

$$\begin{cases} \dfrac{\partial z}{\partial x} = f'(u)u_x = f'\left(\dfrac{x}{y}\right)\dfrac{1}{y} \\ \dfrac{\partial z}{\partial y} = f'(u)u_y = f'\left(\dfrac{x}{y}\right)\left(-\dfrac{x}{y^2}\right) \end{cases}$$

ゆえに,

$$
\begin{aligned}
x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y} &= xf'\left(\frac{x}{y}\right)\frac{1}{y} + yf'\left(\frac{x}{y}\right)\left(-\frac{x}{y^2}\right) \\
&= \frac{x}{y}f'\left(\frac{x}{y}\right) - \frac{x}{y}f'\left(\frac{x}{y}\right) \\
&= 0
\end{aligned}
$$