

$f(x, y) = x^2 - 2xy + 3y^2 - 1$ とおき, (1) で得た公式を用いる. まず, f の第2次偏導関数まで求めると

$$f_x = 2x - 2y = 2(x - y)$$

$$f_y = -2x + 6y = -2(x - 3y)$$

$$f_{xx} = 2$$

$$f_{xy} = -2$$

$$f_{yy} = 6$$

これらを (1) の公式に代入すれば

$$y'' = \frac{2^3(x - 3y)^2 - 2^4(x - y)(x - 3y) + 6 \cdot 2^2(x - y)^2}{(-2)^3(x - 3y)^3}$$

$$\begin{aligned}&= \frac{(x - 3y)^2 - 2(x - y)(x - 3y) + 3(x - y)^2}{(x - 3y)^3} \\&= \frac{2(x^2 - 2xy + 3y^2)}{(x - 3y)^3}\end{aligned}$$