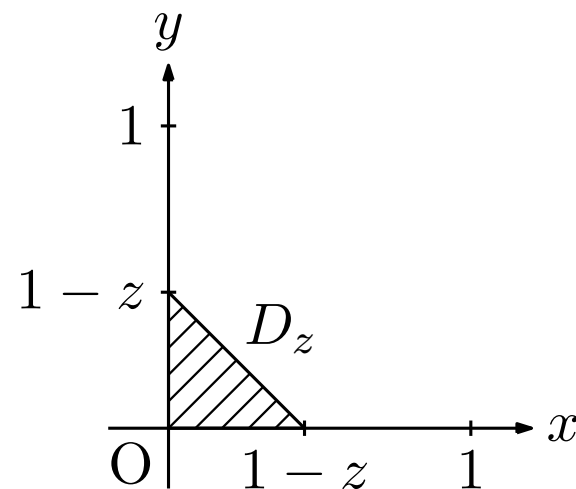
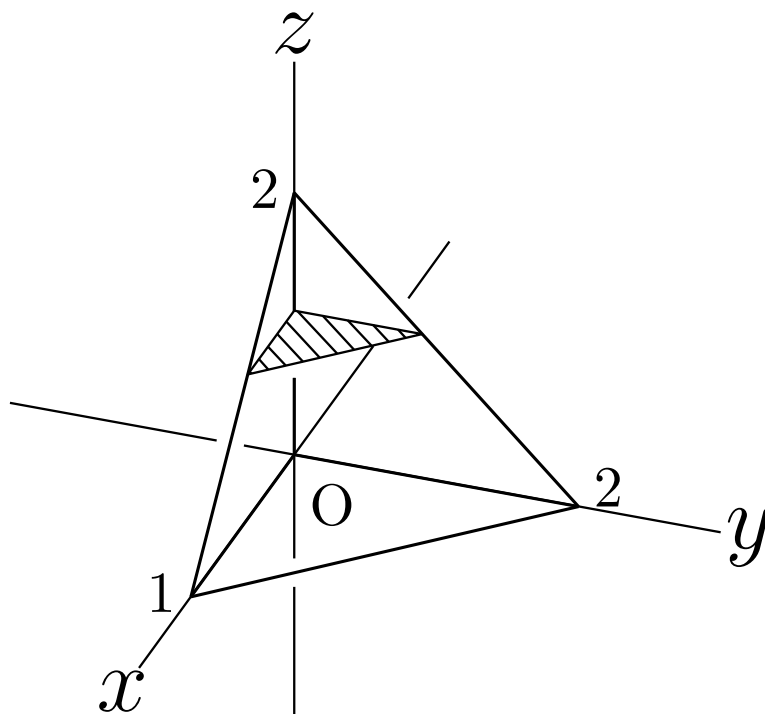


積分領域は左図の四面体で囲まれる部分

D_z は右図の三角形で囲まれる部分



$$\begin{aligned}\text{与式} &= \int_0^1 \left\{ \iint_{Dz} xyz \, dx dy \right\} dz \\ &= \int_0^1 \left\{ \int_0^{1-z} \left\{ \int_0^{1-z-x} xyz \, dy \right\} dx \right\} dz \\ &= \int_0^1 \left\{ \int_0^{1-z} \left[\frac{1}{2} xy^2 z \right]_0^{1-z-x} dx \right\} dz\end{aligned}$$

$$= \frac{1}{2} \int_0^1 \left\{ \int_0^{1-z} x(1-z-x)^2 z \, dx \right\} dz$$

$$1 - z - x = t \text{ とおくと}$$

$$-dx = dt$$

$$x = 1 - z - t$$

$$\begin{array}{c|c} x & 0 \rightarrow 1 - z \\ \hline t & 1 - z \rightarrow 0 \end{array}$$

$$= \frac{1}{2} \int_0^1 \left\{ \int_0^{1-z} (1-z-t)t^2 z \, dt \right\} dz$$

$$\begin{aligned} &= \frac{1}{2} \int_0^1 \left\{ z \int_0^{1-z} \{ (1-z)t^2 - t^3 \} dt \right\} dz \\ &= \frac{1}{2} \int_0^1 \left\{ z \left[\frac{1}{3}(1-z)t^3 - \frac{1}{4}t^4 \right]_0^{1-z} \right\} dz \\ &= \frac{1}{2} \int_0^1 \left\{ z \left\{ \frac{1}{3}(1-z)^4 - \frac{1}{4}(1-z)^4 \right\} \right\} dz \end{aligned}$$

$$= \frac{1}{24} \int_0^1 z(1-z)^4 dz$$

$$1-z=s \text{ とおく と}$$

$$-dz = ds$$

$$z = 1-s$$

z	$0 \rightarrow 1$
s	$1 \rightarrow 0$

$$= \frac{1}{24} \int_0^1 (1-s)s^4 ds$$

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$$= \frac{1}{24} \left[\frac{1}{5} s^5 - \frac{1}{6} s^6 \right]_0^1 = \frac{1}{720}$$