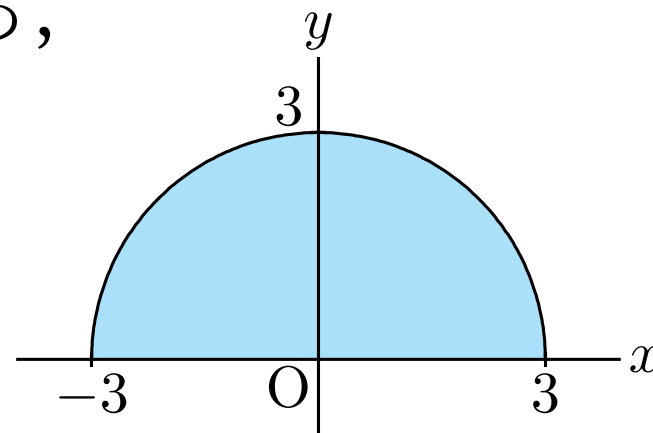


積分領域 D は図の通りだから,

$$D : 0 \leq r \leq \sqrt{3}, 0 \leq \theta \leq \pi$$

$$\begin{aligned} & \iint_D \frac{y}{1+x^2+y^2} dx dy \\ &= \iint_D \frac{r \sin \theta}{1+r^2} \cdot r dr d\theta \end{aligned}$$



$$= \int_0^{\pi} \left\{ \int_0^{\sqrt{3}} \frac{r^2 \sin \theta}{1 + r^2} dr \right\} d\theta$$

$$= \int_0^{\pi} \left\{ \int_0^{\sqrt{3}} \left(1 - \frac{1}{1 + r^2} \right) \sin \theta dr \right\} d\theta$$

$$= \int_0^{\pi} \left[r - \tan^{-1} r \right]_0^{\sqrt{3}} \sin \theta d\theta$$

$$= \int_0^{\pi} \left(\sqrt{3} - \frac{\pi}{3} \right) \sin \theta \, d\theta$$

$$= \left(\sqrt{3} - \frac{\pi}{3} \right) \left[-\cos \theta \right]_0^{\pi} = 2\sqrt{3} - \frac{2}{3}\pi$$