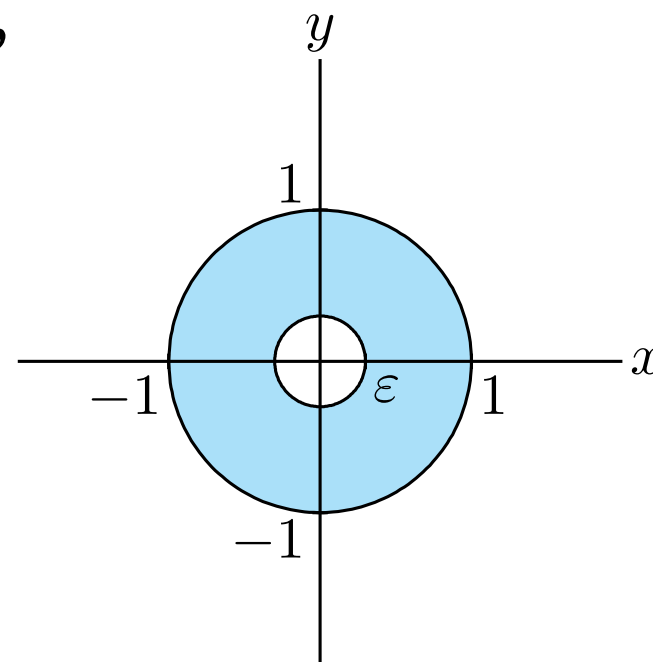


積分領域 D_ε は図の通りだから,

$$D_\varepsilon : \varepsilon \leq r \leq 1, \quad 0 \leq \theta \leq 2\pi$$

$$\begin{aligned} & \iint_{D_\varepsilon} \frac{x^2}{\sqrt{x^2 + y^2}} dx dy \\ &= \iint_{D_\varepsilon} \frac{r^2 \cos^2 \theta}{r} \cdot r dr d\theta \end{aligned}$$



$$\begin{aligned} &= \int_0^{2\pi} \left\{ \int_{\varepsilon}^1 r^2 \cos^2 \theta \, dr \right\} d\theta = \int_0^{2\pi} \left[\frac{1}{3} r^3 \right]_{\varepsilon}^1 \cos^2 \theta \, d\theta \\ &= \int_0^{2\pi} \frac{1 - \varepsilon^3}{3} \cdot \frac{1 + \cos 2\theta}{2} \, d\theta = \frac{1 - \varepsilon^3}{6} \left[\theta + \frac{1}{2} \sin 2\theta \right]_0^{2\pi} \\ &= \frac{1 - \varepsilon^3}{3} \pi \end{aligned}$$

$$\begin{aligned} & \iint_D \frac{x^2}{\sqrt{x^2 + y^2}} dx dy \\ &= \lim_{\varepsilon \rightarrow +0} \iint_{D_\varepsilon} \frac{x^2}{\sqrt{x^2 + y^2}} dx dy \\ &= \lim_{\varepsilon \rightarrow +0} \frac{1 - \varepsilon^3}{3} \pi = \frac{\pi}{3} \end{aligned}$$