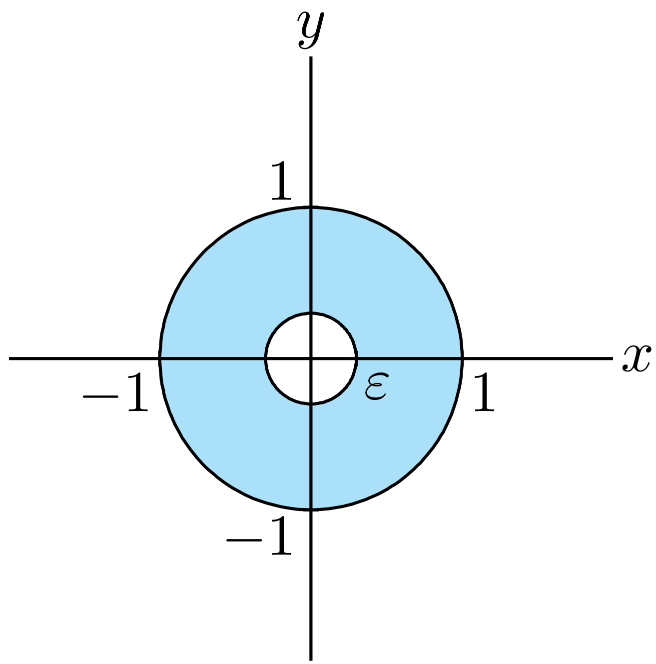
積分領域 D_ε は図の通りだから,



$$D_\varepsilon : \varepsilon \leqq r \leqq 1,\ 0 \leqq \theta \leqq 2\pi$$

$$\iint_{D_\varepsilon} \frac{x^2}{\sqrt{x^2+y^2}}\,dxdy$$

$$= \iint_{D_\varepsilon} \frac{r^2\cos^2\theta}{r} \cdot r\,drd\theta$$

$$= \int_0^{2\pi} \left\{ \int_\varepsilon^1 r^2 \cos^2\theta \, dr \right\} d\theta = \int_0^{2\pi} \left[\frac{1}{3} r^3 \right]_\varepsilon^1 \cos^2\theta \, d\theta$$

$$= \int_0^{2\pi} \frac{1-\varepsilon^3}{3} \cdot \frac{1+\cos 2\theta}{2} \, d\theta = \frac{1-\varepsilon^3}{6} \left[\theta + \frac{1}{2} \sin 2\theta \right]_0^{2\pi}$$

$$= \frac{1-\varepsilon^3}{3} \pi$$

$$\iint_D \frac{x^2}{\sqrt{x^2+y^2}}\,dxdy$$

$$= \lim_{\varepsilon\to+0}\iint_{D_\varepsilon} \frac{x^2}{\sqrt{x^2+y^2}}\,dxdy$$

$$= \lim_{\varepsilon\to+0}\frac{1-\varepsilon^3}{3}\pi = \frac{\pi}{3}$$