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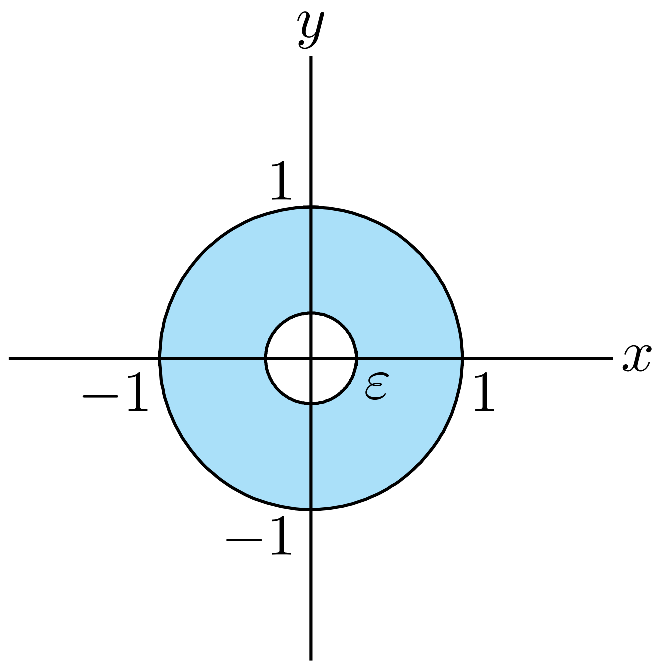
|---|---|---|---|

積分領域 $D_\varepsilon$ は図の通りだから，

$D_\varepsilon : \varepsilon \leqq r \leqq 1,\ 0 \leqq \theta \leqq 2\pi$




$$\iint_{D_\varepsilon} \log(x^2+y^2)\,dxdy$$

$$= \iint_{D_\varepsilon} \log r^2 \cdot r\,drd\theta$$

$$= \int_0^{2\pi} \left\{ \int_\varepsilon^1 2r \log r \, dr \right\} d\theta$$

$$= \int_0^{2\pi} 2 \left\{ \left[ \frac{1}{2} r^2 \log r \right]_\varepsilon^1 - \int_\varepsilon^1 \frac{1}{2} r^2 \cdot \frac{1}{r} \, dr \right\} d\theta$$

$$= \int_0^{2\pi} \left( -\varepsilon^2 \log \varepsilon - \frac{1 - \varepsilon^2}{2} \right) d\theta$$

$$= (-2\varepsilon^2 \log\varepsilon - 1 + \varepsilon^2)\pi$$

$$\iint_D \log(x^2+y^2)\,dxdy = \lim_{\varepsilon\to+0}\iint_{D_\varepsilon} \log(x^2+y^2)\,dxdy$$

$$= \lim_{\varepsilon\to+0}(-2\varepsilon^2 \log\varepsilon - 1 + \varepsilon^2)\pi = -\pi$$

$$\left[\text{ロピタルの定理より，} \lim_{\varepsilon\to+0}\varepsilon^2\log\varepsilon = 0\right]$$