$x = r\cos\theta,\ y = r\sin\theta$ とおくと，$dxdy = rdrd\theta$

(2) の結果より領域 D は $r,\ \theta$ を用いて

$$D : 0 \leqq r \leqq 2\cos\theta, \quad -\frac{\pi}{2} \leqq \theta \leqq \frac{\pi}{2}$$

また，$\sqrt{x^2 + y^2} = \sqrt{r^2\cos^2\theta + r^2\sin^2\theta}$

$$= r$$

$$\begin{aligned}
\iint_D \sqrt{x^2+y^2}\,dxdy &= \iint_D r\cdot rdrd\theta \\
&= \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \left\{ \int_0^{2\cos\theta} r^2\,dr \right\} d\theta \\
&= \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{8}{3}\cos^3\theta\,d\theta
\end{aligned}$$

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$$\begin{aligned}
\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{8}{3}\cos^3\theta\,d\theta &= \frac{8}{3}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} (1-\sin^2\theta)\cos\theta\,d\theta \\
&= \frac{8}{3}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} (\cos\theta - \sin^2\theta\cos\theta)d\theta \\
&= \frac{8}{3}\left[\sin\theta - \frac{1}{3}\sin^3\theta\right]_{-\frac{\pi}{2}}^{\frac{\pi}{2}}
\end{aligned}$$

第2項は置換積分 $t=\sin\theta$ でも得られる.

$$\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{8}{3}\cos^3\theta\, d\theta = \frac{8}{3}\left(2 - \frac{3}{2}\right)$$

$$= \frac{32}{9}$$