Measures of non-compactness in modular spaces and some fixed point theorems

Let X be a metric space and $\mathscr{B}$ the collection of bounded subsets of X. A measure of non-compactness on X is a map $\mu : \mathscr{B} \to [0, +\infty)$ with the properties that (i) $\mu(B) = 0$ if and only if $\overline{B}$ is compact and (ii) $\mu(B) \leq \mu(C)$ if $B \subseteq C$. The concept of measure of non-compactness was first introduced by Kuratowski in 1930. Later in 1955, Darbo used the Kuratowski measure to prove that if T is a continuous self-mapping of nonempty, bounded, closed and convex subset C of a Banach space X satisfying

$$\mu(T(A)) \leq k\mu(A) \qquad \text{for all } A \subseteq C$$

where k is a constant in $(0, 1)$, then T has at least one fixed point in the set C. Darbo's fixed point theorem is a very important generalization of Schauder's fixed point theorem and it includes the existence part of Banach's fixed theorem. Other measures of non-compactness have been defined later, one of the most important is the Hausdorff measure of non-compactness introduced by Goldenstein in 1957.

In this talk we will present the concepts of Kuratowski and Hausdorff measures of non-compactness in modular spaces. Some geometrical concepts associated to those measures of non-compactness such as the uniform non-compact convexity in modular spaces will be recalled. Finally, we will give an outline of some fixed point theorems for multivalued non-expansive mappings.