## CONVEXITY OF HALF SPACES ON A HADAMARD SPACE

## SHUTA SUDO

In this talk, we deal with convexity of two sorts of half spaces on a Hadamard space. A Hadamard space is a nonlinear metric space having a mapping called a geodesic, and it has nonnegative bounded curvature. In a Hadamard space (H, d), for  $x, y \in H$ , a subset

$$\{z \in H \mid d(x, z) \le d(y, z)\}$$

is not convex in general unlike a Hilbert space. However, we can find many examples such a space is convex.

On the other hand, we can define another half space on a Hadamard space as follows:



FIGURE 1

A way to define the above type half space is such as

$$\{z \in H \mid d(x,z)^2 + d(x,y)^2 \le d(y,z)^2\}$$

since

$$\{z \in H \mid ||x - z||^2 + ||x - y||^2 \le ||y - z||^2\} = \{z \in H \mid \langle v - x, y - x \rangle \le 0\}$$

if H is a Hilbert space.

In this work, we consider these two half spaces, and their convexity. We will use a notion of tangent spaces on a Hadamard space to define a half space such as Figure 1.

<sup>(</sup>S. Sudo) DEPARTMENT OF INFORMATION SCIENCE, GRADUATE SCHOOL OF SCIENCE, TOHO UNIVERSITY, MIYAMA, FUNABASHI, CHIBA 274-8510, JAPAN Email address: 7523001s@st.toho-u.ac.jp

Email address. Tozooorbebo.como a.ac.jp

<sup>2020</sup> Mathematics Subject Classification. 47H10, 58C30.

Key words and phrases. Hadamard space; convexity of half spaces.

Topics: Convex analysis, nonlinear analysis